Simulated Annealing for Optimal Ship Routing

O. T. Kosmas[a], D. S. Vlachos[a]

[a]*Laboratory of Computer Sciences,*
Department of Computer Science and Technology,
Faculty of Sciences and Technology, University of Peloponnese
GR-22 100 Tripolis, Terma Karaiskaki, GREECE

Abstract

A simulated annealing based algorithm is presented for the determination of optimal ship routes through the minimization of a cost function. This cost function is a weighted sum of the time of voyage and the voyage comfort (safety is taken into account too). The latter is dependent on both the wind speed and direction and the wave height and direction. The algorithm first discretizes an initial route and optimizes it by considering small deviations which are accepted by utilizing the simulated annealing technique. Using calculus of variations we prove a key theorem which dramatically accelerates the convergence of the algorithm. Finally both simulated and real experiments are presented.

Key words: optimal ship routing, simulated annealing
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1. Introduction

Optimization of ship routing is closely related to both ship characteristics and environmental factors. Ship and cargo characteristics have a significant influence on the application of ship routing. Ship size, speed capability and type of cargo are important considerations in the route selection process prior to sailing and the surveillance procedure while underway. A ship's characteristics identify its vulnerability to adverse conditions and its ability to avoid them [2].

Email addresses: odykosm@uop.gr (O. T. Kosmas), dvlachos@uop.gr (D. S. Vlachos)

On the other hand, environmental factors of importance to ship routing are those elements of the atmosphere and ocean that may produce a change in the status of a ship transit. In ship routing consideration is given to wind, waves, fog and ocean currents. While all of the environmental factors are important for route selection and surveillance, optimum routing is normally considered attained if the effects of wind and waves can be optimized. The effect of wind speed on ship performance is difficult to determine. In light winds (less than 20 knots) ships lose speed in headwinds and gain speed slightly in following winds. For higher speeds, ship speed is reduced in both head and following winds. Wave height is the major factor affecting ship performance. Wave action is responsible for ship motions, which reduce propeller thrust and cause increased drag from steering corrections. The relationship of ship speed to wave direction and height is similar to that of wind. Head seas reduce ship speed, while following seas increase ship speed slightly to a certain point, beyond which they retard it. In heavy seas, the exact performance may be difficult to predict because of the adjustments of the course and speed for ship handling and comfort. Although the effect of sea and swell is much greater than wind, it is difficult to separate the two in ship routing. Fog, while not directly affecting ship performance, should be avoided as much as feasible, in order to maintain normal speed in safe conditions. Although the route may be longer by avoiding fog, transit time may be less due to not having to reduce speed in reduced visibility. In addition, crew fatigue due to increased watch keeping vigilance can be reduced. Ocean currents do not present a significant routing problem, but they can be a determining factor in route selection and diversion. The important consideration to be evaluated are the difference in distance between a great circle route and a route selected for optimum current, with the expected increase of speed of advance from the following current. More details about the effect of environmental factors can be found in [1].

The role of routing is to allocate the available resources to best fulfill certain requirements. In this paper we are interested in the problem of optimal ship routing taking into account only the wave height and direction by using the simulated annealing algorithm [5, 6]. The simulated annealing method is an extension of a Monte Carlo method developed by Metropolis et al [7], to determine the equilibrium states of a collection of atoms at any given temperature T. Since the method was first proposed in [5, 6], much research has been conducted on its use and properties [8, 9, 10, 11, 12, 13, 14, 15]. The method itself is a technique which has attracted significant attention as

suitable for optimization problems of large scale. It can give solution when a desired global extremum is hidden among many, poorer, local extrema. Even though other practical methods have also been found, surprisingly, the implementation of the algorithm is relatively simpler. At the heart of the method is an analogy with thermodynamics, specifically with the way liquids freeze and crystallize, or metals cool and anneal, when they are cooled slowly and thermal mobility is lost. For slowly cooled systems, nature is able to find the minimum energy state.

The paper is organized as follows: in section 2 the formulation of the problem is given. In section 3 we present the method of discrete variational principles and we prove a key result which allows us for an efficient way of searching for the optimal solution. In section 4 the steps of the algorithm are explained and analyzed. Finally in section 5 both simulated and real experiments are performed and the results are presented and discussed.

2. Formulation of the problem

Let us assume that an initial route of a ship is represented by a smooth curve $\vec{r}(s)$ (fig. 1), where the parameter s is the arc length measured from some fixed point A (initial point of the ship route). Then, the tangent vector of the curve of the ship's route in the point of question (see e.g. [16]) is defined as

$$\vec{t} = \frac{\dot{\vec{r}}}{|\dot{\vec{r}}|} = \frac{d\vec{r}}{ds} \tag{1}$$

where $\dot{\vec{r}}$ is the ship's velocity.

We also assume that the moving ship is subject to the influence of the wave height and direction represented by the vector $\vec{w}$ and the wind speed and direction represented by the vector $\vec{v}$.

Under the above assumptions, we define a route cost (a scalar quantity) assigned at every possible route between two points. The route cost includes a weighted combination of the voyage time and the safety (or comfort) of the voyage. The total cost S is given by

$$S = aT + (1 - \alpha)C \tag{2}$$

where T is the total voyage time and C is a scalar characterizing the safety (comfort) of the voyage. The weight α can be tuned by the user depending on his demands. Note here that when $\alpha = 1$, then the only optimization

3

parameter is the voyage time while when $\alpha = 0$, the only optimization parameter is the crew comfort. The scalar C is calculated as a line integral over the route by the following way (up to the linear approximation):

$$C = \int_A^B (\vec{v}^T Z_v + \vec{w}^T Z_w) \vec{t} ds \qquad (3)$$

where $\vec{v}$ is the wind vector, $\vec{w}$ is the wave height vector and Z_v and Z_w are tensors which characterize the ship response to wind and wave, respectively. The calculation of the total voyage time T is a bit more complicated, since both wind and waves can alter the speed of the ship. In general we can write that

$$|r(\dot{\vec{r}})| = u + F(\vec{v}, \vec{w}, \vec{t}) \qquad (4)$$

where u is the speed of the ship in zero wind and F is a function that depends on ship characteristics, wind, wave and direction of the ship movement. For simplicity in the present work, we assume that $F = 0$.

3. Discrete variational principles for mechanical problems

Recently, progress has been made in the development of variational discretization in mechanical problems, both in the fundamental theory and in the applications to challenging problems. Among them, a method of variational integrators based on the discretization of Hamilton's principle has been developed which underlies essentially all of mechanics, from particle mechanics to continuum mechanics. Also the discretization of Lagrange-D'Alembert principle in cases where there is dissipation or external forces present has been used.

Preserving the basic variational structure in the algorithm retains the structure of mechanics (such as conservation laws) at the algorithmic level. This avoids many of the problems with existing integrators, such as spurious dissipation, which may take very expensive runs to eliminate by standard techniques.

With an appropriate development of the connection between mechanics and geometry in the discrete setting, one will be able to use for geometry the technology described in Ref. [17]. The theory of variational integrators is originated from the Hamilton-Jacobi theory and parts of the basic theory have been constructed in Ref. [18] and its numerical analysis is due to various groups, [19, 20].

In the first step of the present work we construct the comfort function C which must be treated with minimization techniques as a basic problem of the calculus of variation. This problem resembles to that appeared in Hamilton's principle where a Lagrangian $L(q, \dot{q})$ is required. Namely to make the integral

$$\delta C = \delta \int_A^B f(q, \dot{q})dt = 0 \tag{5}$$

stationary for all variations δs of s that vanish at the fixed end points, which leads to the Euler-Lagrange equations. The curve obtained by projecting this solution q onto the shape, also solves a variational principle on the shape space. This theory has a PDE counterpart in which one makes a space-time integral stationary which is appropriate for elasticity and fluids for example.

The benefit when the variational integrators derive from the discrete variational mechanics can be shown by reviewing the derivation of the Euler-Lagrange equations and mimic this progress on a discrete level.

3.1. Discretization of the initial ship's route

We consider now the discrete case, where the route is approximated by a polygonal line with edges $\vec{r}_k, k = 0, 1, ... M$. The distance of each line segment is

$$\delta_k = |\vec{r}_k - \vec{r}_{k+1}|, k = 0, 1, ... M - 1 \tag{6}$$

and the tangent vector at each edge is

$$\vec{t}_k = \frac{\vec{r}_k - \vec{r}_{k+1}}{\delta_k}, k = 0, 1, ... M - 1 \tag{7}$$

The cost function can be calculated then as:

$$
\begin{aligned}
T &= \sum_{k=0}^{M-1} \frac{\delta_k}{u} \\
C &= \sum_{k=0}^{M-1} \vec{z}_k^T \vec{t}_k \delta_k \\
S &= aT + (1 - a)C
\end{aligned}
\tag{8}
$$

where

$$\vec{z}_k^T = (\vec{v}^T Z_v + \vec{w}^T Z_w) \tag{9}$$

calculated at point $\vec{r}_k$ and at time

$$T_k = \sum_{j=0}^{k-1} \frac{\delta_k}{u} \qquad (10)$$

Consider now a small change in $\vec{r}_k$ by $\epsilon \vec{x}_\phi$, where ϵ is a small positive number and $\vec{x}_\phi$ is the unit vector with angle ϕ with the horizontal axis. The new route will have cost S' with

$$
\begin{aligned}
\Delta S &= S' - S \\
&= a\epsilon(\vec{z}_k^T - \vec{z}_{k-1}^T)\vec{x}_\phi + (1-a)\epsilon \frac{(\vec{t}_k^T - \vec{t}_{k-1}^T)\vec{x}_\phi}{u} \\
&= \epsilon \left(a(\vec{z}_k^T - \vec{z}_{k-1}^T) + (1-a)\frac{\vec{t}_k^T - \vec{t}_{k-1}^T}{u} \right)\vec{x}_\phi \qquad (11)
\end{aligned}
$$

where we have assumed that

$$|\vec{r} + \vec{\lambda}| \approx |\vec{r}| + \frac{\vec{r}\vec{\lambda}}{|\vec{r}|} \ , \ |\vec{\lambda}| \ll |\vec{r}| \qquad (12)$$

3.2. Searching using simulated annealing

Consider now the initial route in Fig. 1. The route is broken into several line segments, the number of which depends on the solution of the wind and wave forecasts. In this way a route is represented as a list of way-points. Initially the total cost of the route is calculated. Then in every iteration step every way-point of the route is moved by an elementary length, perpendicular to the line which connects the departure and the arrival points. This elementary length is specified by the resolution of the wind and the wave forecasts. Every movement has a positive or negative contribution to the total cost. A movement is accepted even if it has a positive contribution to the total cost with a probability which depends on the temperature of the system, the so-called Boltzmann probability distribution,

$$Prob(E) \approx exp(-E/kT) \qquad (13)$$

expressing the idea that the system in thermal equilibrium at temperature T has its energy probabilistically distributed among all different energy states E. Even at low temperature, there is a change, albeit very small, of a system being in a high energy state. Therefore, there is a corresponding change

6

for the system to get out of a local energy minimum in favour of finding a better, more global one. The quantity k (Boltzmann's constant) is a constant of nature that relates temperature to energy. In other words, the system sometimes goes uphill as well as downhill, but the lower the temperature, the less likely is any significant uphill excursion.

Initially the temperature is high, but as the algorithm proceeds, the temperature is decreased to zero. At this point, only movements with negative contributions are accepted. This method is known as simulated annealing and is used to avoid the local minima. Notice here that there is no systematic way to decide if the calculated route is the optimal one. In most cases however, the voyage time is very critical and thus the optimal route is close to the shortest initial one. The only case that was observed in our experiments, where the iterative method was locked in a local minimum, was when the line between the departure and arrival points was very close to a small obstacle (fig. 2). In this case the method could not overcome the obstacle and the algorithm was terminated. The reason is that only continuous transformations of the route are performed in every step of the algorithm. To overcome this difficulty, we consider several initial routes, as it will be explained in the next section.

To accelerate the convergence of the searching mechanism, it would be useful to know, in which directions, the cost function is more sensitive to the transformations of the initial route. Fortunately we can prove that when minimizing the cost function for any segment of the ship's route, the resulting movement is parallel or anti-parallel to the direction of the wave propagation. The proof proceeds as follows (considering only the contribution of the wave to the cost function). The cost function is given

$$C = \int_A^B (\vec{w}^T Z_w)\vec{t}ds \tag{14}$$

and we assume a change to the route close to point $\vec{r}(s_0)$, which results in a new tangent vector of the form

$$\vec{t'} = \vec{t} + \epsilon\delta(s - s_0)\vec{x}_\phi \tag{15}$$

where δ is the Dirac function and $\epsilon, \vec{x}_\phi$ are defined in section 3.1. The variation in the cost function will be

$$\delta C = \int_A^B (\vec{w}^T Z_w)\epsilon\delta(s - s_0)\vec{x}_\phi ds \tag{16}$$

7

or

$$\delta C = \epsilon \vec{z}_{|s=s_0} \vec{x}_\phi \qquad (17)$$

where $\vec{z}_{|s=s_0}$ is the vector $\vec{w}^T Z_w$ calculated at the point of the route where $s = s_0$. It is obvious now that the maximum change of the cost will happen if the angle between the vector $\vec{z}_{|s=s_0}$ and $\vec{x}_\phi$ is 0 or π. The acceleration now of the convergence is obtained by selecting variations of the initial route only in the above directions. In our experiments, the direction of the vector $\vec{z}_{|s=s_0}$ is always the same as the direction of the wave propagation, hence the accepted variations of the initial route are those which are parallel or antiparallel to the direction of the wave.

4. Description of the algorithm

For the initialization of the algorithm, we have to calculate one or several initial routes. In the case that there are no obstacles between the starting and ending points, the initial route is simply the largest cycle joining these two points. On the other hand, if there is an obstacle between the starting and ending point, then there are two possible initial routes, each of which bypasses the obstacle from a different side. It is a good practice to start with shortest initial routes (one route for every possible bypass), thus having already optimized the voyage time. In this case it is obvious that, if point B belongs to the shortest path between points A and C, then the shortest path between points B and C is part of the shortest path between points A and C. It is proved in [2] that the shortest path bypassing an obstacle is tangent to the convex hull of the obstacle. So the problem of calculating the shortest path, is reduced to the calculation of the shortest path from the contact point of this tangent to the ending point. Recursively we get that the number of initial paths is 2^n, where n is the number of obstacles between the starting and ending points. The algorithm must check all the initial routes to decide for the optimal solution even if the computational time is increased.

As soon as one initial route is calculated, we have to decide for the number of way-points that will be used to represent the route. We can think this number as the degree of a polynomial used to fit a given smooth curve. Increasing the degree of the polynomial results in a better fitting. However at the same time there are at least three reasons to argue that this not exactly the case here. The first one is that increasing the number of way-points, the computational time is increased and the actions that have to be taken by the

ship's crew are increased too. The second reason concerns the period T_d of the forecast data. If u is the speed of the ship, then nothing is known about the drift of the environmental data in the time interval T_d and thus in the space interval $u \cdot T_d$. The last reason is the details of the model used to forecast the environmental data, and more precisely the mesh that is used. If L_d is the distance of the mesh points, then the ship for the time interval L_d/u is moved in constant environmental parameters. Experiments performed with various numbers of way-points showed that an optimal selection for the distance of the way-points should be close to L_d. It is then a good practice to continuously adjust the number of way-points. If the distance between two successive way-points is greater than D_M, another way-point is added between them. On the other hand, if two successive way-points are closer than a special distance D_m , the two way-points are merged into one. In the experiments that will be presented in this paper

$$D_M = 2 \cdot L_d \ , \ D_m = L_d/2$$

The calculation now of the optimal route is performed as follows:

1. Consider the set $\{\vec{r}_k, k = 0, (1), M\}$ of $M+1$ points describing the initial route.
2. Choose a random k (between 1 and $M-1$, i.e. including all points of the set except for the end points $k = 0$ and $k = M$) and generate a random real ϕ such that $0 \le \phi \le 2\pi$.
3. Calculate the difference of the cost function between the initial route and the route resulting by shifting $\vec{r}_k$ by $\epsilon \vec{x}_\phi$, where ϵ is a small real number and $\vec{x}_\phi$ is a unit vector in the chosen random direction.
4. Generate a random number p between 0 and 1 and accept the variation of the route if $p < e^{-\Delta S/E}$. If the variation is not accepted, goto to step 6.
5. Check $\vec{r}_k$ with the adjacent points. This means that, if two points are very close, we merge them or if they are too far, we add a new point between them.
6. Decrease the temperature E of the system in analogy with the cooling of the system (simulated annealing). Check if the algorithm has to terminate, otherwise goto step 2.

The symbols used above represent the following: p stands for the probability Boltzmann distribution and E is the temperature of the algorithm. Other symbols have been explained previously.

5. Numerical Results

5.1. Constant wave field

In the first experiment (fig. 3), the optimal route for a uniform (a) or piecewise uniform (b,c) wave field is calculated. In the first experiment, the direction of the wave was upwards and does not change through the voyage. Notice that the way-points are moved upwards in order to maximize the product $|\vec{u} \cdot \vec{w}|$, where $\vec{u}$ is the velocity of the ship. This is very reasonable, since the higher moments around the ship's axis are developed when the wave is perpendicular to the ship's direction. The same behavior is observed in fig. 3.b and fig. 3.c where the magnitude of the wave field is constant but the direction is inverted once and twice respectively.

5.2. The effect of parameter α

The effect of the parameter α is studied in the next experiment (fig. 4). In this experiment, the wave field is constant with direction perpendicular to the line joining the starting (S) and ending (E) point of the voyage. When the parameter α takes its largest value (1), no action is taken from the algorithm take into account the comfort of the voyage. Since voyage time is the only parameter to optimize, the calculated value is simply the largest circle joining the starting and ending points.

By decreasing the value of the parameter α, the comfort of the voyage starts to act cumulatively to the total cost function, thus making the algorithm to turn the ship parallel or antiparallel to the direction of the wave field. This tendency is compensated by the time of the voyage which is increased. By decreasing more the value of the parameter α, the contribution of the voyage time to the total cost is decreased, leading to longer routes which are aligned with the direction of the wave field.

5.3. Field test

In the next experiment, the optimal route is calculated from the port of Thessaloniki ($40.5197N$, $22.9709E$) to the port of Ag. Nikolaos ($35.1508N$, $25.7227E$). The forecast data are taken from the *POSEIDON* system which uses floating buoys for real time measurements and mathematical models to predict the wave characteristics for the next 48 hours [3]. In this experiment we have to take into account several initial routes, since there are obstacles (islands) each of which doubles the number of total initial routes [2]. The initial routes are plotted in fig. 5 with the dominant wave direction across

the routes. There are three main regions, the first one has dominant waves coming from the north (this the region close to Thessaloniki), the second has dominant waves coming from North-East (the middle region) and the third has dominant waves coming from South-West (this the region close to A. Nikolaos).

Fig. 6 shows the calculated optimal route (solid line) with the one calculated by the application of genetic algorithms (dotted line). The value of the parameter α was 0.5. The two routes agree except for a small part in the beginning of the voyage which is caused by the property of the route calculated with the proposed algorithm to be transformed continuously from the initial route. Nevertheless the difference in the total cost of the two routes is negligible. An interesting trend of the calculated route, is that the ship is trying to "*hide*" behind the obstacles (islands) where the magnitude of the wave height is decreased dramatically.

6. Summary and conclusions

An effective operational algorithm for the calculation of optimal ship routing has been developed. The algorithm is based on the simulated annealing technique to search for an optimal solution. The searching process is accelerated by taking into account a key result, which enables us to take into account only the variations of the initial route which are parallel or antiparallel to the direction of the wave field. Field tests have been performed which exhibit the efficiency of the proposed algorithm by comparing the calculated routes with those calculated by exhaustive and time consuming genetic algorithms.

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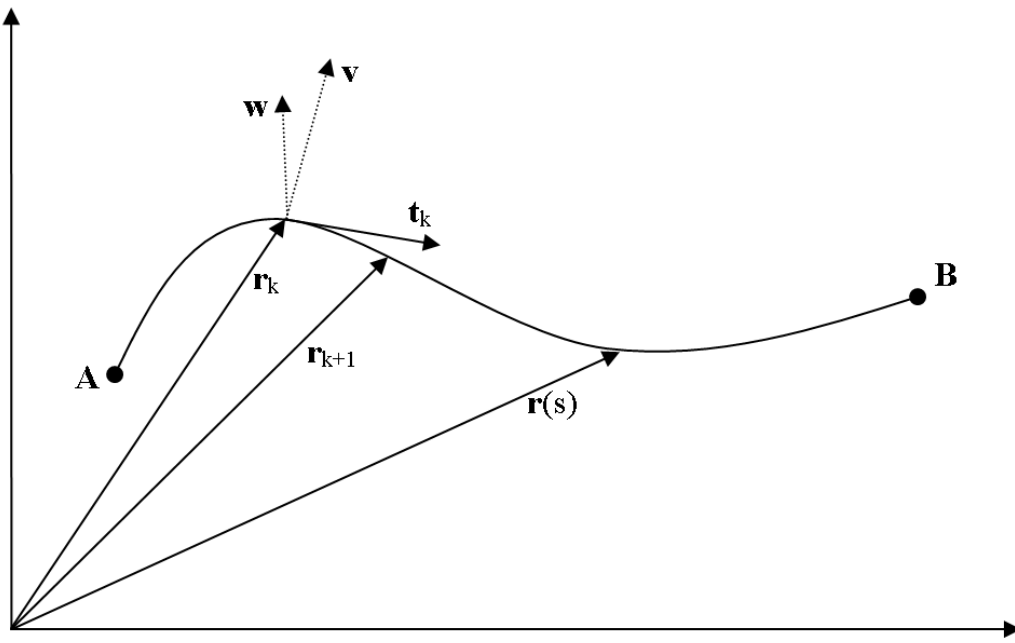
Figure 1: A route from point A to point B. $r(s)$ is the parametrization of the route, and w, v are the wave and wind vectors respectively.
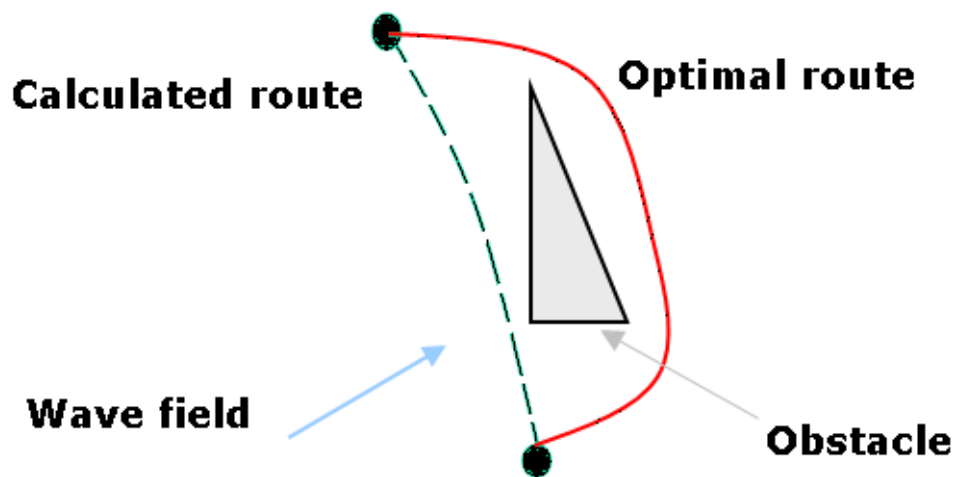
Figure 2: The solution can be locked in a local minimum if the optimal route cannot be generated by continuous transformations of the initial route.
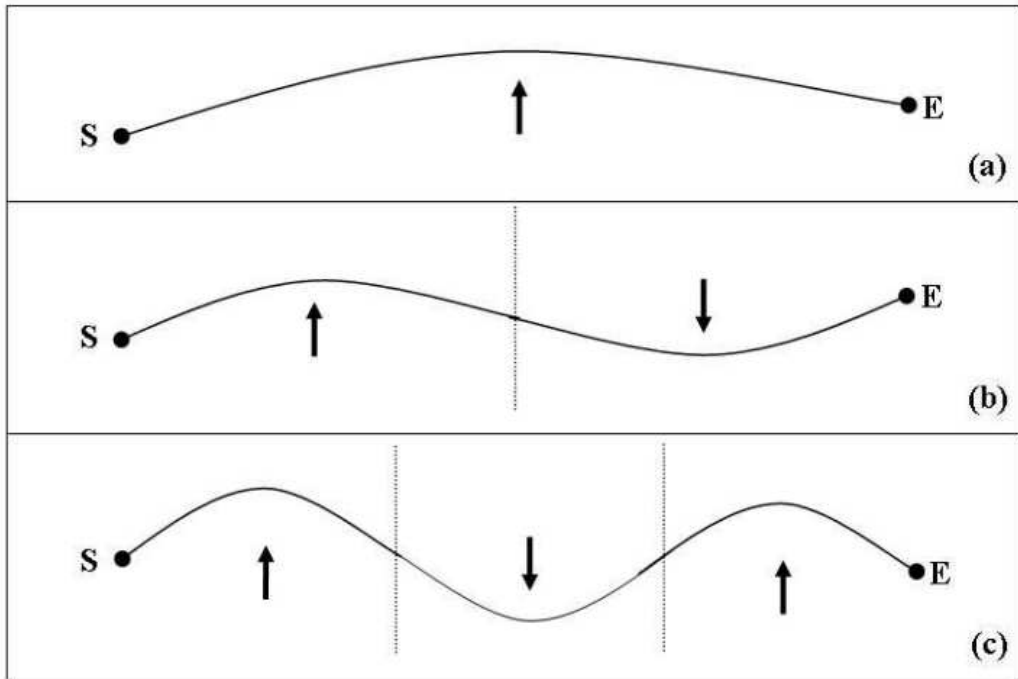
Figure 3: Optimal route calculated in (a) a wave field with constant magnitude and direction and (b),(c) in a wave field with constant magnitude but with direction which is inverted once and twice respectively during the voyage.
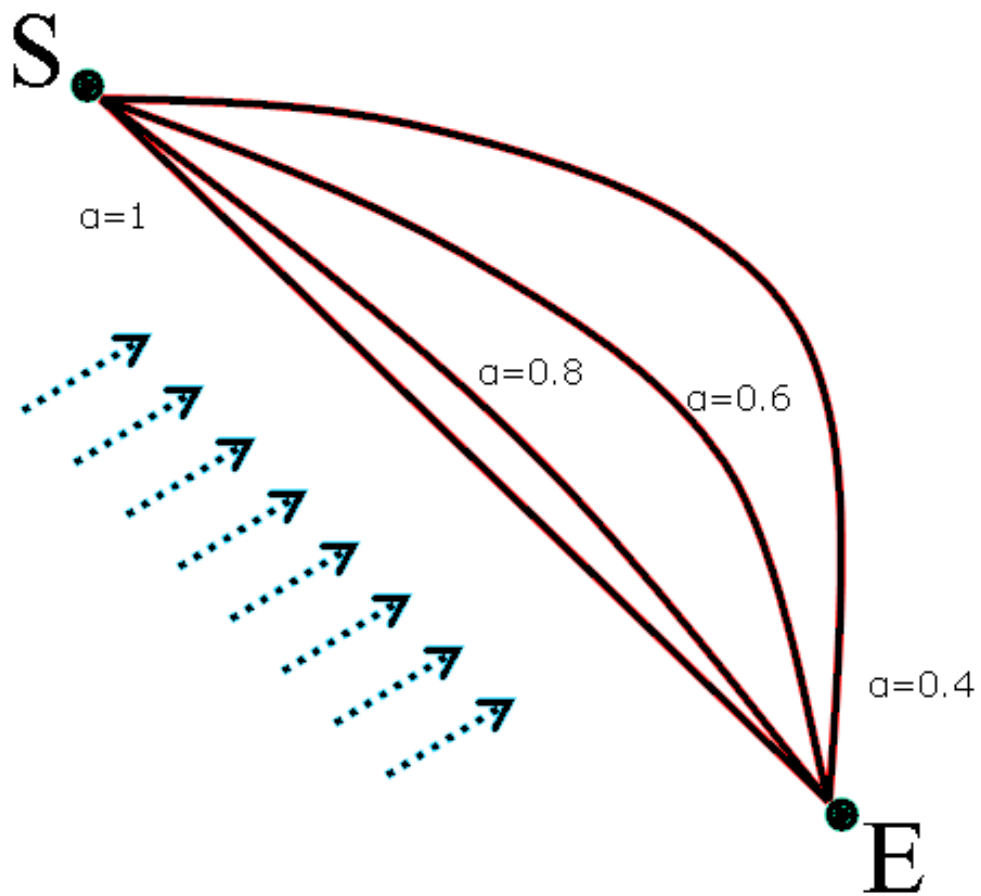
Figure 4: The optimal route for various values of α. $\alpha = 1$ represents the route with minimum route time, while smaller values of α represent a more safe (comfort) voyage.
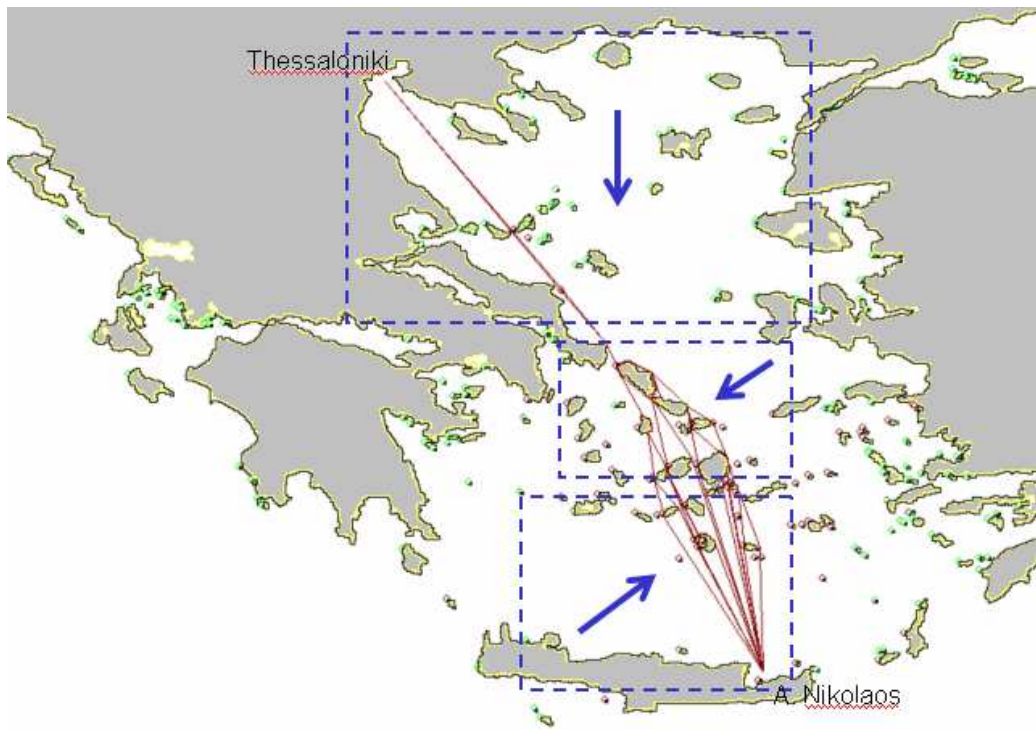
Figure 5: Initial routes calculated from the port of Thessaloniki to the port of A. Nikolaos. The dominant wave fields are shown across the routes with an arrow showing the direction and magnitude of the wave field.

Figure 6: Optimal route calculated from the port of Thessaloniki to A. Nikolaos for the case shown in fig. 5 with the proposed algorithm (solid line) and with the application of genetic algorithms (dotted line).